Supplemental Information for The Domestic Politics of Strategic Retrenchment, Power Shifts, and Preventive War

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I Preliminaries

Lemma I.1. $\tau^{*A} = 1$

Proof. Note that τ^{*A} solves

$$\max_{\tau} \theta \bar{y} + \rho x^{*}(\tau).$$

The first order conditions are $\frac{d\rho\pi(\tau)}{d\tau} = 0$ and hence $\rho\pi'(\tau^{*A}) = 0$ in any interior solution. By assumption, $\rho \neq 0, \pi' > 0, \pi'' < 0$, which implies $\tau^{*A} = 1$. □

Lemma I.2. Let τ^{*B} be faction B's most preferred tax rate. Then τ^{*B} satisfies

$$\pi'(\tau^{*B}) = \frac{(1-\theta)\bar{y}}{1-\rho}. \qquad \text{(SI-I.1)}$$

Proof. Let τ^{*B} be faction B's most preferred tax rate. Then τ^{*B} solves

$$\max_{\tau}(1-\tau)(1-\theta)\bar{y} + (1-\rho)(\pi(\tau) - c),$$

the first order conditions of which are $(1-\theta)\bar{y} = (1-\rho)\pi'(\tau^{*B})$. Hence

$$\pi'(\tau^{*B}) = \frac{(1-\theta)\bar{y}}{1-\rho}$$

in an interior solution.[1] □

Lemma I.3.

[1]Note: If $\pi'(\tau^{*B}) < \frac{(1-\theta)\bar{y}}{1-\rho}$, $\forall \tau$, then $\tau^{*B} = 0$. Likewise, if $\pi'(\tau^{*B}) > \frac{(1-\theta)\bar{y}}{1-\rho}$, $\forall \tau$, then $\tau^{*B} = 1$.

- $\frac{\partial u_B}{\partial \tau} = (1-\rho)\pi'(\tau) - (1-\theta)\bar{y}$

- $\frac{\partial u_B}{\partial \theta} = ((1-\rho)\pi'(\tau) - (1-\theta)\bar{y})\frac{\partial \tau}{\partial \theta} - (1-\tau)\bar{y}$

- $\frac{\partial u_B}{\partial \rho} = ((1-\rho)\pi'(\tau) - (1-\theta)\bar{y})\frac{\partial \tau}{\partial \rho} - (\pi(\tau)+c)$

- $\frac{\partial u_B}{\partial \bar{y}} = ((1-\rho)\pi'(\tau) - (1-\theta)\bar{y})\frac{\partial \tau}{\partial \bar{y}} + (1-\tau)(1-\theta)$

- $\frac{\partial u_B}{\partial \mu^H} = ((1-\rho)\pi'(\tau) - (1-\theta)\bar{y})\frac{\partial \tau}{\partial \mu^H}$

Further,

$$\frac{\partial u_B}{\partial \tau} \begin{cases} < 0 & \text{if } \pi'(\tau) < \frac{(1-\theta)\bar{y}}{1-\rho} \\ = 0 & \text{if } \pi'(\tau) = \frac{(1-\theta)\bar{y}}{1-\rho} \\ > 0 & \text{if } \pi'(\tau) > \frac{(1-\theta)\bar{y}}{1-\rho} \end{cases}$$

Proof. Partial derivatives are obtained by direct calculation, using the total derivative and implicit differentiation, treating τ as a function of $\theta, \rho, \bar{y}, \mu^H$, respectively. □

II Analysis of the two period game

By backwards induction, conditional on reaching period 2 (which requires the absence of both war and domestic revolt in period 1), equilibrium behavior in period 2 is given by $\bar{\mu}, \underline{\mu}, \hat{\tau}$ and $x^*(\hat{\tau})$ as in Section 3.3. It will be convenient to define $u_B(\tau)$ to be the contemporaneous utility to faction B of not revolting, when the tax rate is τ, given that war will not occur in the current period.

Faction B does not revolt to tax rate τ_1^H in period 1 when

$$(1+\delta)(1-\mu^H)\bar{y} \leq u_B(\tau_1^H) + \delta[(1-q)(1-\mu^H)\bar{y} + q(1-\rho)(\pi(1)+c)], \tag{SI-II.1}$$

where $u_B(\tau) = (1-\tau)(1-\theta)\bar{y} + (1-\rho)(\pi(\tau)+c)$. The left-hand side of inequality SI-II.1 is the discounted payoff to B for revolting in period 1 and the right-hand side is the utility of accepting the tax rate in period 1 and continuing to period 2.

Solving SI-II.1 with equality and rearranging, let $\hat{\tau}_1^H$ solve

$$u_B(\hat{\tau}_1^H) = (1+\delta q)(1-\mu^H)\bar{y} - \delta q(1-\rho)(\pi(1)+c). \tag{SI-II.2}$$

The above establishes the tax rate in period one, when the cost of revolt is μ^H for which faction B is indifferent between revolt and no revolt in period one, essentially "rolling the dice" by allowing the game to continue to day two. This tax rate in period one, $\hat{\tau}_1^H$, reflects the possibility that on day two the cost of revolt will be μ^L, in which case faction B obtains its *least* preferred tax rate. Alternatively, if the game has not ended in period 1 and the cost of revolt is μ^H in period 2, faction A sets tax rate $\hat{\tau}$ (see equation 3.4, main text)

For certain costs of revolt (as shown in section 3.3), faction B's revolt decision is insensitive to the tax policy. When the cost of revolt in period one is small enough (relatively), revolting is a best response to any tax that faction A sets. When the cost of revolting is high enough, faction A can set its most preferred tax rate, τ^{*A}, and faction

B will not revolt. For an intermediate range of costs of revolt, M, faction B may be appeased by A in both periods one and two (see *Supporting Information* for the formal definition of M).

Lemma II.1. For $\mu \in M$ there exists $\tau^{*B} < \hat{\tau}_1^H < \hat{\tau}$ as SPNE offers of faction A.

Proof. The proof is in two parts. First, that $\hat{\tau}$ and $\hat{\tau}_1^H$ exist and that $\hat{\tau} > \hat{\tau}_1^H$ is shown. Second, that faction B does not revolt in period 1 when the state is μ^H and does not revolt to tax rate $\hat{\tau}$ in period 2 when the state is μ^H as part of a SPNE is shown.

Existence of $\hat{\tau}$ and $\hat{\tau}_1^H$ follows from definition of $\underline{\mu}, \bar{\mu}, \underline{\mu}^1, \bar{\mu}^1$ and equations 3.4 and SI-II.2. That $\hat{\tau} > \hat{\tau}_1^H$ follows from observing that (i) u_B is decreasing in τ for $\tau > \tau^{*B}$ (Lemma I.3), (ii) $\hat{\tau} > \tau^{*B}$ and $\hat{\tau}_1^H > \tau^{*B}$ for $\mu \in M$ and (iii) $(1+\delta q)(1-\mu)\bar{y} - \delta q(1-\rho)(\pi(1)+c) < (1-\mu)\bar{y}$ for $\mu \in M$. That is: $\mu^H \in M$ implies $\mu^H > \underline{\mu}^1$ which implies $\hat{\tau} > \hat{\tau}_1^H$, when $\hat{\tau}$ exists.

Finally, by construction of $\hat{\tau}$ and $\hat{\tau}_1^H$ (equations 3.4 and SI-II.2), faction B is indifferent between revolt and not in when the cost of revolt is μ^H in periods 1 and 2 respectively.

□

Substantively, Lemma II.1 establishes a range of costs of revolt for which tax rates born by faction B increase over time. In period 2, the cost of revolt may be μ^L, in which case revolt is not credible and faction A sets taxes at its most preferred level. Alternatively, the cost of revolt in period 2 may be μ^H, in which case the equilibrium tax rate, $\hat{\tau}$, is greater than $\hat{\tau}_1^H$. This lemma is helpful for identifying how "rapid and sudden" increases in military strength need be to generate international commitment problems.

II.I Revolution constraints, period 1

For certain costs of revolution, faction B would never revolt in period 1. To establish precisely when this is the case, some lemmata are in order.

Lemma II.2.

1. $u_B(\cdot)$ is strictly concave

2. $\arg\max_{\tau\in[0,1]} u_B(\tau) = \tau^{*B}$.

3. $\max_{\tau\in[0,1]} u_B(\tau) = (1-\underline{\mu})\bar{y}$.

Proof. Part one follows from the strict concavity of π: $\frac{\partial^2 u_B}{\partial\tau^2} = (1-\rho)\pi''(\tau) < 0$ for all $\tau \in [0,1]$. Part two follows from the definition of τ^{*B} and part three follows from part two and the definition of $\underline{\mu}$.

□

By Lemma II.2, for $\hat{\tau}_1^H$ to exists, the right hand side of equation SI-II.2 must be bounded above and below. Specifically, if $min\{u_B(0), u_B(1)\} \leq (1+\delta q)(1-\mu)\bar{y} - \delta q(1-\rho)(\pi(1)+c) \leq (1-\underline{\mu})\bar{y}$, then there exists at least one solution to SI-II.2 and further, if $max\{u_B(0), u_B(1)\} \leq (1+\delta q)(1-\mu)\bar{y} - \delta q(1-\rho)(\pi(1)+c) < (1-\underline{\mu})\bar{y}$ then there exists a $\hat{\tau}_1^H$ satisfying SI-II.2 with $\hat{\tau}_1^H > \tau^{*B}$. Obviously, as faction A's utility is increasing in the tax rate (subject to no revolt), in equilibrium faction A offers the largest $\hat{\tau}_1^H$ satisfying equation SI-II.2.

Let $\underline{\mu}^1$ solve

$$(1+\delta q)(1-\underline{\mu}^1)\bar{y} - \delta q(1-\rho)(\pi(1)+c) = (1-\underline{\mu})\bar{y}. \tag{SI-II.3}$$

For example if $\mu < \underline{\mu}^1$, faction B would revolt to any tax rate in period 1. If $\mu > \underline{\mu}^1$ and $\mu < \underline{\mu}$ then faction B could be appeased in period 1 but will revolt to any tax rate in period 2.

Let $\bar{\mu}^1$ solve

$$(1+\delta q)(1-\bar{\mu}^1)\bar{y} - \delta q(1-\rho)(\pi(1)+c) = \max\{(1-\theta)\bar{y} + (1-\rho)(\pi(0)+c), (1-\rho)(\pi(1)+c)\}. \tag{SI-II.4}$$

If $\mu^H > \bar{\mu}^1$, then any offered tax rate in period 1 would would result in faction B revolting.[2]

Hence, letting $\min\{\bar{\mu}, \bar{\mu}^1\} > \mu > \max\{\underline{\mu}, \underline{\mu}^1\}$, ensures there exists $\hat{\tau}$ and $\hat{\tau}_1^H$ with $\hat{\tau} > \hat{\tau}_1^H > \tau^{*B}$. To that end, let the interval of costs of revolution for which faction B can be

[2]This does not completely describe equilibria tax rates, as equation SI-II.2 can be satisfied when $(1+\delta q)(1-\mu)\bar{y} - \delta q(1-\rho)(\pi(1)+c) \in [\min\{u_B(0), u_B(1)\}, \max\{u_B(0), u_B(1)\}]$. In this case either $\tau^{*B} \geq \hat{\tau} \geq \hat{\tau}_1^H$ or $\hat{\tau} \geq \tau^{*B} \geq \hat{\tau}_1^H$.

appeased in in equilibriumto be

$$M = (\max\{\underline{\mu}, \underline{\mu}^1\}, \min\{\bar{\mu}, \bar{\mu}^1\}).$$

That is, M is an interval for which we are guaranteed increasing tax rates offered by faction A are accepted by faction B, when revolt is credible. A sufficient condition for M to be non-empty is given in the next Lemma.

Lemma II.3. If $\bar{y} > 1 + \delta q$ then $\max\{\underline{\mu}, \underline{\mu}^1\} < \min\{\bar{\mu}, \bar{\mu}^1\}$.

Proof. Clearly $\bar{\mu} > \underline{\mu}$ and $\bar{\mu}^1 > \underline{\mu}^1$. It can easily be shown that $\bar{\mu} > \underline{\mu}^1$, so it is sufficient to show that $\bar{\mu}^1 > \underline{\mu}$. To see this, note that $\bar{\mu}^1$ solves the equation

$$(\bar{y}(1 + \delta q) - \delta q(1 - \rho)(\pi(1) + c)) - \bar{y}(1 + \delta q)\mu = \max\{u_B(0), u_B(1)\} \tag{SI-II.5}$$

and

$\underline{\mu}$ solves

$$\bar{y}(1 - \mu) = u_B(\tau^{*B}) \tag{SI-II.6}$$

both of which are linear in μ as $u_B(\tau^{*B})$ can be regarded as a constant not depending on μ.

Comparing the linear equations SI-II.5 and SI-II.6, it can be seen that the intercept of SI-II.6 is greater than the intercept of equation SI-II.5 iff $\bar{y} > (1-\rho)(\pi(1)+c)$. Further, the slope of both are negative, so that the line defined in SI-II.6 is steeper than SI-II.5 iff $\bar{y} > 1 + \delta q$. Note that $1 + \delta q > 1 > (1 - \rho)(\pi(1) + c)$. By Lemma II.2, $u_B(\tau^{*B}) \geq \max\{u_B(0), u_B(1)\}$, so that $\bar{y} > 1 + \delta q$ guarantees $\bar{\mu}^1 > \underline{\mu}$.

□

III Proofs of results in text

Proof. [Proof of Proposition 3.1] By Lemmas I.1 and I.2, and noting that π is increasing and concave, the result follows. □

Proof. [Proof of Lemma 3.2] State 2 rejects offer $(1-x)$ when $1-\pi(\tau)-c > 1-x$, in which case faction A receives $\pi(\tau) - c$. Hence faction A gets $\pi(\tau) - c$ if it offers $x > \pi(\tau) + c$, and gets x if it offers $x \leq \pi(\tau) + c$. As $c > 0$, faction A offers $x^* = \pi(\tau) + c$, and state 2 is indifference between accepting the $(1 - x^*)$ and rejecting the offer, inducing war. □

Proof. [Proof of Proposition 3.4]

$\mu \in M$ assures that there exist tax offers to which faction B would not revolt (Lemma II.1). State 2 accepts an offer in period one if the continuation value of accepting $x^*(\hat{\tau}_1^H)$ (assuming no revolt) does not exceed the value of going to war:

$$(1 + \delta)(1 - \pi(\hat{\tau}_1^H)) - c \geq 1 - x^*(\hat{\tau}_1^H) + \delta\left((1 - q)(1 - \pi(\hat{\tau}) - c) + q(1 - \pi(1) - c)\right),$$

where the left hand side is the value of war in period 1 to state 2 and the right hand side is comprised of the value of accepting offer x^* in period one plus the expected value of period-two bargains, discounted by δ. Simplifying, the result obtains. □

III.I Comparative statics

Recall that $\mu^L = 1$ by assumption, to that when the cost of revolt is high there is no credible threat of revolution. Thorough this section assume $\mu^H \in M$ so that $\hat{\tau}$ and $\hat{\tau}_1^H$ are defined by

$$(1-\mu^H)\bar{y} = u_B(\hat{\tau}). \tag{3.4}$$

and

$$(1+\delta)(1-\mu^H)\bar{y} = u_B(\hat{\tau}_1^H) + \delta(1-q)(1-\mu^H)\bar{y} + \delta q(1-\rho)(\pi(1)+c), \tag{SI-II.2}$$

respectively, where $u_B(\tau) = (1-\tau)(1-\theta)\bar{y} + (1-\rho)(\pi(\tau)+c)$.

Lemma III.1. $\pi'(\hat{\tau}) < \frac{(1-\theta)\bar{y}}{1-\rho}$ and $\pi'(\hat{\tau}_1^H) < \frac{(1-\theta)\bar{y}}{1-\rho}$ for $\mu^H \in M$

Proof. If $\mu^H \in M$ then by Lemma II.1 $\hat{\tau} > \hat{\tau}_1^H > \tau^{*B}$. But τ^{*B} satisfies $u_B'(\tau^{*B}) = 0$, so that $\pi'(\tau^{*B}) = \frac{(1-\theta)\bar{y}}{1-\rho}$. As $\pi'' < 0$, we have $\pi'(\hat{\tau}) < \frac{(1-\theta)\bar{y}}{1-\rho}$. Clearly, $\pi' > 0$, $\pi'' < 0$, $\hat{\tau} > \hat{\tau}_1^H$ and $\pi'(\hat{\tau}) < \frac{(1-\theta)\bar{y}}{1-\rho}$ jointly imply $\pi'(\hat{\tau}_1^H) < \frac{(1-\theta)\bar{y}}{1-\rho}$

□

Proposition III.2. Let $\mu^H \in M$ and $\hat{\tau}$ and $\hat{\tau}_1^H$ be as in eqns. 3.4 and SI-II.2.

$$\frac{\partial \hat{\tau}}{\partial \theta} = \frac{(1-\hat{\tau})\bar{y}}{(1-\rho)\pi'(\hat{\tau}) - (1-\theta)\bar{y}} \tag{SI-III.1}$$

$$\frac{\partial \hat{\tau}_1^H}{\partial \theta} = \frac{(1-\hat{\tau}_1^H)\bar{y}}{(1-\rho)\pi'(\hat{\tau}_1^H) - (1-\theta)\bar{y}} \tag{SI-III.2}$$

$$\frac{\partial \hat{\tau}}{\partial \rho} = \frac{\pi(\hat{\tau}) + c}{(1-\rho)\pi'(\hat{\tau}) - (1-\theta)\bar{y}} \tag{SI-III.3}$$

$$\frac{\partial \hat{\tau}_1^H}{\partial \rho} = \frac{\delta q(\pi(1) + c)}{(1-\rho)\pi'(\hat{\tau}_1^H) - (1-\theta)\bar{y}} \tag{SI-III.4}$$

$$\frac{\partial \hat{\tau}}{\partial \bar{y}} = \frac{(1-\mu^H) - (1-\hat{\tau})(1-\theta)}{(1-\rho)\pi'(\hat{\tau}) - (1-\theta)\bar{y}} \tag{SI-III.5}$$

$$\frac{\partial \hat{\tau}_1^H}{\partial \bar{y}} = \frac{(1+\delta q)(1-\mu^H) - (1-\hat{\tau}_1^H)(1-\theta)}{(1-\rho)\pi'(\hat{\tau}_1^H) - (1-\theta)\bar{y}} \tag{SI-III.6}$$

$$\frac{\partial \hat{\tau}}{\partial \mu^H} = \frac{\bar{y}}{(1-\theta)\bar{y} - (1-\rho)\pi'(\hat{\tau})} \tag{SI-III.7}$$

$$\frac{\partial \hat{\tau}_1^H}{\partial \mu^H} = \frac{(1+\delta q)\bar{y}}{(1-\theta)\bar{y} - (1-\rho)\pi'(\hat{\tau}_1^H)} \tag{SI-III.8}$$

Proof. By implicit differentiation, and Lemma I.3, the partial derivatives are calculated. □

Proof. [**Proof of Theorem 3.5**]

Let $NWC := (1-q)\pi(\hat{\tau}) - \pi(\hat{\tau}_1^H)$. We proceed by determining the sign of $\frac{\partial NWC}{\partial g}$, for $g \in \{\theta, \rho, \mu^H\}$. Note that

$$\frac{\partial NWC}{\partial g} = (1-q)\pi'(\hat{\tau})\frac{\partial \hat{\tau}}{\partial g} - \pi'(\hat{\tau}_1^H)\frac{\partial \hat{\tau}_1^H}{\partial g},$$

for $g \in \{\theta, \rho, \mu^H, c\}$.

Inequality, θ:

From Proposition 3.4, preventive war is increasing in θ whenever $\frac{\partial NWC}{\partial \theta} > 0$. Now,

$$\frac{\partial NWC}{\partial \theta} = \frac{(1-q)(\pi'(\hat{\tau})(1-\hat{\tau})\bar{y}}{(1-\rho)\pi'(\hat{\tau}) - (1-\theta)\bar{y}} - \frac{(1-\hat{\tau}_1^H)(\bar{y})\pi'(\hat{\tau}_1^H)}{(1-\rho)\pi'(\hat{\tau}_1^H) - (1-\theta)\bar{y}}.$$

By Lemma III.1, $(1-\rho)(\pi'(\hat{\tau}) - (1-\theta)\bar{y}) < (1-\rho)(\pi'(\hat{\tau}_1^H) - (1-\theta)\bar{y}) < 0$. Hence, $(1-\hat{\tau}_1^H)\bar{y}\pi'(\hat{\tau}_1^H) > (1-q)\pi(\hat{\tau})(1-\hat{\tau})\bar{y}$ is sufficient for $\frac{\partial NWC}{\partial \theta} > 0$. By Lemma II.1, $\hat{\tau}_1^H < \hat{\tau}$ and hence $1 > \frac{(1-q)\pi'(\hat{\tau})}{\pi'(\hat{\tau}_1^H)}$ is sufficient for $\frac{\partial NWC}{\partial \theta} > 0$. But $q < 1$ by assumption and $\pi'(\hat{\tau}) < \pi'(\hat{\tau}_1^H)$ by $\pi' > 0$ and $\pi'' < 0$ and Lemma II.1.

Sharing rule, ρ: From Proposition 3.4, preventive war is increasing in ρ whenever $\frac{\partial NWC}{\partial \rho} > 0$. Now,

$$\frac{\partial NWC}{\partial \rho} = \frac{(1-q)\pi'(\hat{\tau})(\pi(\hat{\tau}) + c)}{(1-\rho)\pi'(\hat{\tau}) - (1-\theta)\bar{y}} - \frac{\pi'(\hat{\tau}_1^H)\delta q(\pi(1) + c)}{(1-\rho)\pi'(\hat{\tau}_1^H) - (1-\theta)\bar{y})}.$$

Because $(1-\rho)(\pi'(\hat{\tau})-(1-\theta)\bar{y} > (1-\rho)(\pi'(\hat{\tau}_1^H)-(1-\theta)\bar{y} > 0$, (and because $\pi' > 0$)

$$\frac{\delta q}{1-q} > \frac{\pi'(\hat{\tau})}{\pi'(\hat{\tau}_1^H)}$$

is sufficient for $\frac{\partial NWC}{\partial \rho} > 0$. Because $\pi'(\hat{\tau}) < \pi'(\hat{\tau}_1^H)$, $\delta > (1-q)/q$ is sufficient for $\frac{\partial NWC}{\partial \rho} > 0$. As $q > 1/2$, by assumption, the result follows.

<u>Cost of revolt, μ^H</u>: From Proposition 3.4, preventive war is increasing in μ^H whenever $\frac{\partial NWC}{\partial \mu^H} > 0$. Now,

$$\frac{\partial NWC}{\partial \mu^H} = \frac{(1-q)\pi'(\hat{\tau})\bar{y}}{(1-\theta)\bar{y}-(1-\rho)\pi'(\hat{\tau})} - \frac{\pi'(\hat{\tau}_1^H)(1+\delta q)\bar{y}}{(1-\theta)\bar{y}-(1-\rho)\pi'(\hat{\tau}_1^H)}.$$

Again, because $(1-\theta)\bar{y}-(1-\rho)(\pi'(\hat{\tau}) > (1-\theta)\bar{y}-(1-\rho)(\pi'(\hat{\tau}_1^H) > 0$, it is the case that $(1-q)\pi'(\hat{\tau})\bar{y} < \pi'(\hat{\tau}_1^H)(1+\delta q)\bar{y}$ is sufficient for $\frac{\partial NWC}{\partial \mu^H} < 0$. But $(1-q)/(1+\delta q) < 1$, so the result follows.

<u>Cost of war, c</u>: From Proposition 3.4, preventive war is decreasing in c whenever $\frac{\partial NWC}{\partial c} < 1$. However,

$$\frac{\partial NWC}{\partial c} = \frac{(1-q)(1-\rho)}{(1-\theta)\bar{y}-(1-\rho)\pi'(\hat{\tau})} - \frac{\pi'(\hat{\tau}_1^H)(1+\delta q)(1-\rho)}{(1-\theta)\bar{y}-(1-\rho)\pi'(\hat{\tau}_1^H)}$$

but $(1-q)(1-\rho) < (1+\delta q)(1-\rho)$ and $(1-\theta)\bar{y}-(1-\rho)(\pi'(\hat{\tau}) > (1-\theta)\bar{y}-(1-\rho)(\pi'(\hat{\tau}_1^H)$ so in fact $\frac{\partial NWC}{\partial c} < 0$.

$\square$